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Chemical Engineering Journal 86 (2002) 61–67

Chemical Engineering Journal

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A theoretical explanation for the inversion temperature

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Abstract

When comparing superheated steam and dry air in a drying process a temperature has been found, at which the drying rates into the gases compared are equal. This temperature is called the inversion temperature. Many different values for the inversion temperature have been reported in the literature in the last years. This paper presents a model to calculate the inversion temperature. The model is based on the equations describing coupled heat and mass transfer and the calculated results are in good agreement with experimental findings without the need of fitting parameters to the data. The range of inversion temperatures found can be explained with the theory of the model.

Keywords: Superheated steam; Airless drying; Inversion temperature; Heat and mass transfer

1. Introduction

Superheated steam has been known as a drying gas since the turn of the century. Using superheated steam in the drying process can lead to substantial energy savings, if the vapour evaporating from the product is condensed and if the heat of condensation is recovered and used elsewhere in the overall process. Therefore, superheated steam drying is an attractive drying process for industrial use especially if the cost of energy for the drying is a major part of the cost of manufacture.

Early research on the drying of different materials with air, humid air and superheated steam yielded contradictory results concerning the change in drying rate with the different gas phase compositions. Thus, some investigations concentrated on the evaporation of water into air at different temperatures and humidities. Yoshida and Hyodo [1] carried out extensive experiments using a wetted wall column and were the first to find an inversion temperature at which the evaporation rates were equal independent of the gas composition. They initially assumed that at this temperature the evaporation rate of water becomes independent of the humidity of the gas for a constant gas mass flow at the inlet. Since then, a number of publications have dealt with the theoretical and experimental determination of this inversion temperature, revealing a dependence of the inversion temperature on the gas compositions being compared. Table 1 summarises some of the results of these investigations.

At very high moisture contents of the material to be dried the drying process is mainly governed by unhindered evaporation. In most cases the drying gas is a mixture of air and the vapour of a pure evaporating liquid. This significantly simplifies the problem, because the gas can be considered as a binary mixture. Consequently, the problem can be described by the equations for coupled heat and mass transfer. In this paper, these equations are set up and solved for a range of gas temperatures and compositions. The calculations are carried out at atmospheric pressure using air as the inert gas and water as the evaporating liquid, which represents a common case in drying. The results of these calculations are the evaporation rates.

The evaporation rates into pure superheated steam and completely dry air are found to be equal at a certain temperature which is most commonly called the inversion temperature. This definition of the inversion temperature implies that the amount of evaporation is so small that no change in gas temperature and composition in the direction of the gas flow is incured. Therefore its use is limited to this theoretical case. In industrial processes the surface areas encountered are not infinitely small and the change in gas conditions in the direction of the gas flow cannot be neglected. Therefore, new and more precise definitions for the inversion temperature are necessary.

Corresponding to the classical definition of the inversion temperature, the temperature, at which the evaporation rates from an infinitely small surface area into the two gas flows with vapour mole fractions a and b are equal, is defined as the local inversion temperature $\mathrm{LIT}_{a,b}$. If the surface areas are not infinitely small (as in all experimental investigations

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PII: S1385-8947(01)00273-X

Nomenclature

A	area of interface (m^2)
c_p	molar heat capacity (J/mol/K)
c_t	molar density (mol/m^3)
d	diameter of the wetted wall column (m)
D	diffusion coefficient (m^2/s)
h	molar enthalpy (J/mol)
L	length from the beginning of the plate (m)
$\dot{n}$	molar flux ($mol/m^2/s$)
$\dot{N}$	molar flow (mol/s)
Nu	Nusselt number
p_{sat}	saturation pressure (Pa)
Pr	Prandtl number
$\dot{q}$	heat flux (W/m^2)
Re	Reynolds number
Sc	Schmidt number
Sh	Sherwood number
T	temperature (K)
X	mole fraction subscript variable for local inversion temperature (mol/mol)
y	mole fraction (mol/mol)

Greek symbols

α	heat transfer coefficient ($W/m^2/K$)
β	mass transfer coefficient (m/s)
Δh_v	latent heat of vaporisation (J/mol)
Φ_W	Ackermann correction factor

Subscripts

A	air
Conv	convection only L liquid
Evap	evaporated out at outlet
G	bulk flow gaseous phase total (convection and radiation)
i	component i
I	interface
in	at inlet
v	vapour, vaporisation

cited) and gas conditions change in the direction of the gas flow, an apparent inversion temperature is found. This apparent inversion temperature $AIT_{a,b}$ is the gas temperature at the inlet to the evaporation surface, for which the evaporation rates averaged over the whole area into the two gas flows with the inlet vapour mole fractions a and b are equal. Any apparent inversion temperature is only valid for the technical system regarded and therefore has to be accompanied by detailed information about the geometry and gas flow conditions of this system.

Chow and Chung [2,3] undertook the first theoretical approach to describe the inversion temperature. Their findings are the basis of the most common explanation for the existence of the inversion temperature. The wet bulb depression becoming smaller with increasing steam content leads to a smaller heat flux and, consequently, to a smaller evaporation rate. At higher temperatures this effect becomes small compared to the total temperature difference and is compensated by the changing thermodynamic properties for heat transfer in steam. Sheikoleslami and Watkinson [10] and Faber et al. [11] addressed the problem by using an equilibrium approach. The assumption of the gas being saturated at the outlet makes the description of the mass transfer obsolete. For a more comprehensive explanation of the inversion temperature the coupled heat and mass transfer has to be taken into account.

Although most publications determine the inversion temperature on the basis of constant mass flow at the inlet a comparison on the basis of constant volumetric flow, i.e. constant gas velocity, is of theoretical and practical interest as well. Faber et al. [11] reported an inversion temperature of 406 °C for constant volumetric flow which is considerably higher than their value of 160 °C found for constant mass flow.

2. Theoretical background

The evaporation of a liquid into a gas flow of a mixture of its own vapour and an inert gas is a problem of coupled

Table 1
Inversion temperatures from various investigations [12]

Reference	Theory/experiment	Description	$T_{inversion}$ (°C)
Here	Theory	Wetted wall column, turbulent	199
Yoshida and Hyodo [1]	Experiment	Wetted wall column, turbulent	160–176
Chow and Chung [2]	Theory	Flat plate, laminar	250
Chow and Chung [3]	Theory	Flat plate, turbulent	190
Here	Theory	Flat plate, turbulent	200
Haji and Chow [4]	Experiment	Flat plate, turbulent	170–220
Hasan et al. [5]	Theory	Moving flat surface and gas, laminar	230–260
Al-Taleb et al. [6]	Theory	Moving flat surface, laminar	225
Wu et al. [7]	Theory	Wedge, turbulent	200
Ramamurthy [8]	Experiment	Impinging jets, laminar	218–230
Bond et al. [9]	Experiment	Impinging jets, turbulent	175
Sheikoleslami and Watkinson [10]	Theory	Equilibrium approach	164
Faber et al. [11]	Both	Fluidised bed, equilibrium approach	160

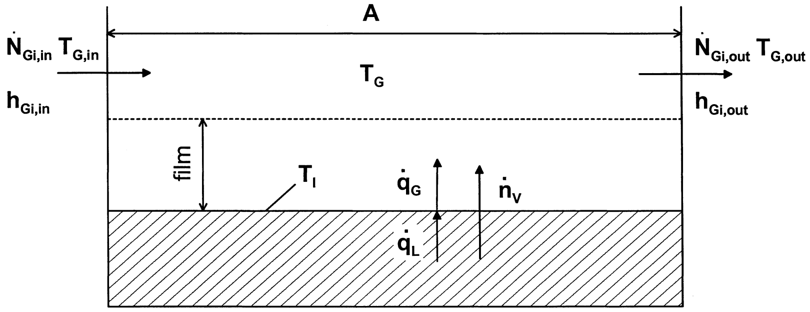

Fig. 1. Increment of a liquid surface exposed to a gas flow.

heat and mass transfer. The problem regarded here is easy to describe since it only deals with a pure liquid and a gas that can be considered as a binary mixture.

Fig. 1 shows a very small increment of a liquid surface exposed to a gas flow. The gas temperature and composition can be assumed to be constant within this increment in the direction of the gas flow. The coupled heat and mass transfer can be described by applying the film theory. This leads to a set of two transfer and two balance equations.

(1) The heat transfer equation corrected for simultaneous mass transfer by the analytically derived Ackermann correction [13]:

$$\dot{q}_G=\alpha_G \frac{\Phi_W}{\exp(\Phi_W)-1}(T_I - T_G) \text{ and } \Phi_W=\frac{\dot{n}_V c_{pv}}{\alpha_G} \quad (1)$$

(2) The mass transfer equation:

$$\dot{n}_v = c_t \beta_G \ln\left(\frac{1-y_{vI}}{1-y_{vG}}\right) \quad (2)$$

(3) The heat balance for the gaseous phase:

$$\dot{q}_G A = (\dot{N}_{Ga,in} c_{pa} + \dot{N}_{Gv,in} c_{pv})(T_{G,out} - T_{G,in}) + \dot{n}_v c_{pv}(T_{G,out} - T_I) A \quad (3)$$

(4) With the assumptions that

- heat is transferred from the gas to the liquid by convection only;
- there is no heat flux between the wall and the liquid;
- the liquid phase is at the temperature of evaporation.

The heat balance at the interface:

$$\dot{q}_L = 0 = \dot{n}_v \Delta h_v(T_I) + \dot{q}_G \quad (4)$$

The equilibrium at the interface determines the saturation pressure p_{sat} and the interface gas composition y_{vI} at the interface temperature. All thermodynamic properties of the vapour and the inert gas are functions of temperature and pressure. Assuming an ideal gas mixture, its properties can be calculated from the pure components with knowledge of the partial pressures. The unknown variables are the outlet temperature of the gas $T_{G,out}$, the interface temperature T_I, the evaporative flux $\dot{n}_v$ and the heat flux $\dot{q}_G$ which can be obtained by solving the set of non-linear equations described above.

The thermodynamic properties at atmospheric pressure needed for air and water are calculated with equations from Daubert and Danner [14], the diffusion coefficient is estimated using the method of Fuller et al. [15] recommended in Reid et al. [16]. More detailed information about the calculations and the computer code can be found in Schwartze [12]. If the Nusselt and Sherwood equation for the geometrical system regarded are known, the evaporation rates can be calculated with no further experimental data sources. For the calculation of evaporation rates a wetted wall column similar to the one used in the investigation of Yoshida and Hyodo [1] and a flat plate resembling the experimental set-up of Haji and Chow [4] are chosen. A single increment is considered. The liquid phase is assumed to be stationary. The required heat and mass transfer coefficients are calculated with the correlations of Krupiczka and Rotkegel [17] (wetted wall column)

$$Nu = \frac{\alpha_G d}{\lambda_G} = 0.0279\, Re^{0.791}\, Pr^{0.44} \quad (5)$$

$$Sh = \frac{\beta_G d}{D_{av}} = 0.0279\, Re^{0.791}\, Sc^{0.44} \quad (6)$$

and Whitaker [18] (flat plate)

$$Nu = \frac{\alpha_G 1}{\lambda_G} = 0.036\, Re^{0.8}\, Pr^{0.43} \quad (7)$$

$$Sh = \frac{\beta_G 1}{D_{av}} = 0.036\, Re^{0.8}\, Sc^{0.43} \quad (8)$$

For pure superheated steam there is no mass transfer resistance and the evaporation is solely controlled by the heat transfer to the interface. Eq. (2) does not apply and the interface temperature is determined by the pressure of the system.

Up to now the existence of the inversion temperature has been explained by the temperature depression between the interface and the bulk gas flow, as well as the change of thermodynamic properties with increasing temperature (e.g. Chow and Chung [2,3]). Consequently, only the heat transfer was taken into consideration. A more comprehensive explanation of the inversion temperature phenomenon is possible by considering the coupled heat and mass transfer.

In air the process of evaporation is governed by simultaneous heat and mass transfer. The driving force for the heat transfer process is the temperature difference across the boundary layer. The driving force for the mass transfer is the vapour concentration difference across the boundary layer. This boundary layer incorporates a resistance for both transport processes. At low temperatures the process is mainly heat transfer controlled since the concentration difference across the boundary layer is large compared with a rather small temperature difference. With increasing gas temperature the temperature difference and consequently the heat transfer rate, increase. The concentration difference increases as well because of the combined increase in temperature and saturation pressure at the interface. While the increase of the temperature difference is unlimited, the concentration difference reaches a maximum when the interface saturation pressure reaches the pressure of the system. Consequently, the whole process of evaporation becomes mass transfer controlled and the mass transfer resistance is the governing factor.

In pure superheated steam the mass transfer resistance does not exist, since the gas consists only of the evaporating species. This explains why for high temperatures the evaporation into pure superheated vapour is higher than into dry air. The disadvantages of the higher interfacial temperature and less heat transfer potential in superheated steam are outweighed by the absence of a mass transfer resistance.

3. Results and discussion

3.1. Local inversion temperature

As mentioned above the inversion temperature is most commonly known as the temperature at which the evaporation rates into pure superheated steam and completely dry air are equal. This includes the implicit prerequisite of infinitely small increments so as to get a negligible change in gas composition and temperature in the direction of the gas flow. In the new definition this is called the local inversion temperature $\mathrm{LIT}_{0,1}$. For the calculation of the local inversion temperature a single increment of the geometry has to be considered.

Calculations have been performed with constant gas mass flow for a wetted wall column similar to the experimental set-up of Yoshida and Hyodo [1] and for a flat plate resembling the experimental set-up of Haji and Chow [4]. The length of the increment to meet the requirement of negligible change in gas conditions for both symmetries was 10^{-3} m. The results of the calculations for the wetted wall column are shown in Fig. 2. The local inversion temperature $\mathrm{LIT}_{0,1}$ can be found at the intersection of the evaporation rate curves for completely dry air and pure superheated steam. For the calculated geometry $\mathrm{LIT}_{0,1}$ is 198.6 °C and calculations with mass flow rates of 9100 and 27,300 kg/m^2/h show that it is independent of the gas mass flow. The local inversion temperature $\mathrm{LIT}_{0,1}$ found for one increment of the flat plate is 198.4 °C and as well independent of the gas mass flow. In this case the Reynolds number incorporates the distance of the increment from the beginning of the flat plate. Calculations showed that the chosen distance affected the result for the local inversion temperature only in the second decimal place. These results imply that the geometry of the system does not have a large influence on the inversion temperature found.

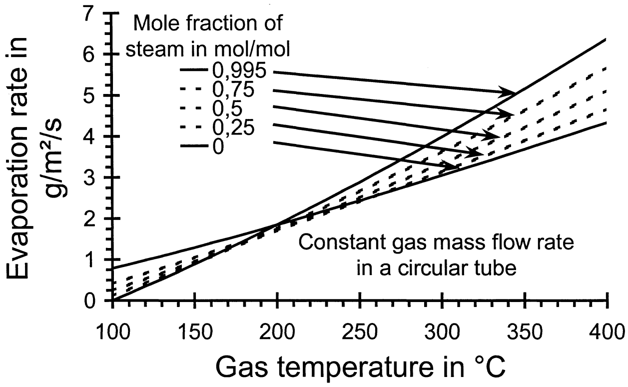

Fig. 2. Evaporation rates vs. gas inlet temperature at constant gas mass flow [12].

There is no gas temperature for which the evaporation rate is independent of the gas composition. This is in contrast to the statement of Yoshida and Hyodo [1] and supports the findings of other investigators (e.g. Chow and Chung, [2,3]). Different local inversion temperatures $\mathrm{LIT}_{a,b}$ will be found if other gas compositions are compared.

Fig. 3 shows the range of local inversion temperatures possible for the wetted wall column. The limiting temperatures approached are approximately 155 °C for $\mathrm{LIT}_{X\to 1,1}$ (humid

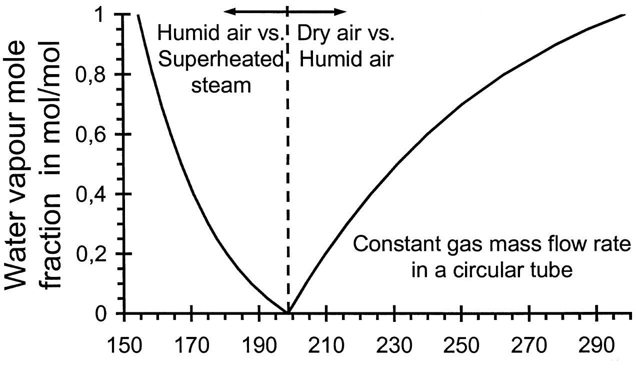

Fig. 3. Local inversion temperature for different gas compositions being compared (dry air, humid air, superheated steam).

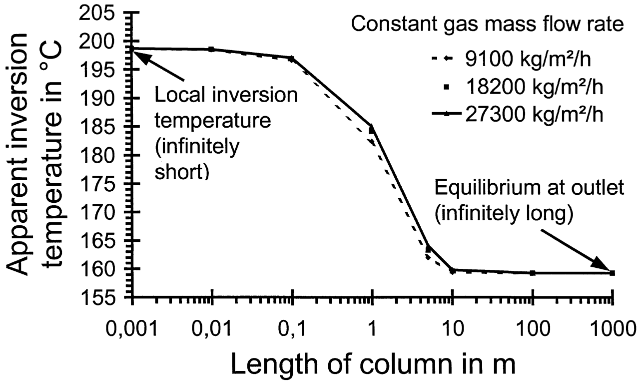

Fig. 4. Change in apparent inversion temperature $AIT_{0,1}$ with increasing length of the column at constant gas mass flow.

air versus superheated steam) and 300 °C for $LIT_{0,1-x\rightarrow 1}$ (dry air versus humid air) with $0 < X < 1$. The proposed notation is able to precisely describe the whole range of possible local inversion temperatures.

3.2. Apparent inversion temperature

The local inversion temperature is a theoretical value. Experimental investigations will always face a finite amount of evaporation and thus a change in gas composition and temperature in the direction of the gas flow, which cannot be neglected. The inversion temperature found in real systems thus will be an apparent inversion temperature. This apparent inversion temperature has to be defined at the inlet condition of the gas flow, because this is the only point where air can actually be completely dry. Fig. 4 illustrates the influence of the amount of evaporation and thus the length of the liquid surface in the direction of the gas flow, on the apparent inversion temperature $AIT_{0,1}$ for the wetted wall column. The calculations have been performed incrementally with increment lengths of 10^{-3} m.

The range of apparent inversion temperatures found is determined by the local inversion temperature on one end and by an infinitely long column with the maximum possible change in gas composition in the direction of the flow on the other end. This maximum change is defined by a complete saturation of the gas at the outlet and therefore resembles the results of the equilibrium approach (Sheikoleslami and Watkinson, [10], Faber et al. [11]). Depending on the length of the column regarded the $AIT_{0,1}$ will be found between 198.6 °C ($LIT_{0,1}$) and 159.3 °C. In between these two theoretically defined limits the gas mass flow has an influence on the observed inversion temperature. It affects the changes in gas temperature and composition along the column. With a higher gas flow the humidification of the dry air will be less pronounced and consequently the apparent inversion temperature will be higher than that for smaller gas mass flows. However, this effect is small compared to the overall drop in the apparent inversion temperature with increasing column lengths and thus gas condition changes. For an infinitely long column with saturated gases at the outlet the apparent inversion temperature is again independent of the gas mass flow. In real evaporation processes, such as drying in the constant rate period, more evaporation in the drier at constant gas mass flow will result in lower apparent inversion temperatures.

All local and apparent inversion temperatures shown above have been calculated with constant gas mass flows at the inlet. If these inversion temperatures are calculated on the basis of constant gas volumetric flows (i.e. gas velocities) at the inlet, the temperature values found are significantly higher. Due to the different densities of air and water vapour the gas mass flow of air is higher at a constant velocity of the gases. The $LIT_{0,1}$ on the basis of constant gas volumetric flow was calculated to be at 388.4 °C for the wetted wall column.

3.3. Comparison with experimental results

More calculations were performed simulating the experiments of Yoshida and Hyodo [1] as well as those of Haji and Chow [4]. The results were compared with the experimental data to validate the calculation model. Fig. 5 shows a comparison between the calculations and the experimental data. The results are within ±20% of the experimental data. The agreement is very good considering that no parameters are fitted to the experimental data. The model used is able to describe the real process of evaporation of a liquid into a gas flow of its own vapour and a non-transferring gas.

Due to the assumption of a perfectly insulated liquid-wall interface, the neglected radiative heat transfer by steam and - for the wetted wall column, the neglected movement of the liquid phase, the majority of the calculated evaporation rates show an underprediction of the experimental data. The influence of the radiative heat transfer can be estimated from the data corrected for purely convective heat transfer given by Yoshida and Hyodo [1]. As expected the agreement with the calculated results is even better. For the flat plate only two values for the evaporation rate are explicitly given in the

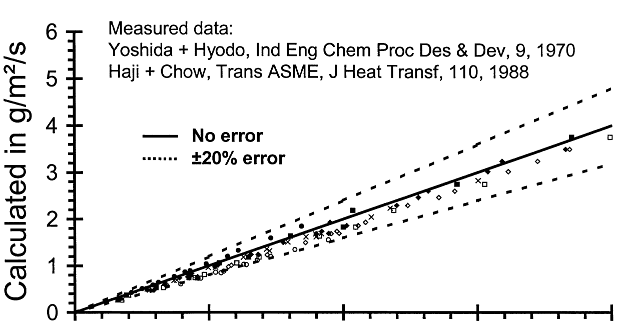

Fig. 5. Comparison between the calculations and experimental data of Yoshida and Hyodo [1] and Haji and Chow [4] from [12].

original publication (Haji and Chow, [4]). All other values have been extracted from graphs that showed a line fitted to all the experimental data. The largest underprediction is found for the evaporation rates into dry air measured by Haji and Chow [4], which, in their publication, have been corrected for the influence of conductive heat transfer from the walls to the liquid. Fig. 5 includes only uncorrected, measured data since no details of the correction procedure were given. This leads to a less satisfactory agreement. The corrected values are only slightly smaller than the evaporation rates calculated in this investigation.

4. Conclusion

When comparing drying with superheated steam and with dry air, a temperature is found where the evaporation rates into both gases are equal. This temperature is most commonly referred to as the inversion temperature. Many different values for the inversion temperature have been reported. By describing the process in terms of coupled heat and mass transfer the interface temperature, evaporation rate and gas outlet conditions for a small increment can be calculated. The incremental procedure makes the description of transfer from finite surface areas possible without loss of accuracy. The inversion temperature can be found at the intersection of the evaporation rate curves for dry air and superheated steam when plotted over the gas inlet temperature. The inversion temperatures calculated for a wetted wall column and a flat pate are well within the range of inversion temperatures reported in the literature. It is shown that the inversion temperature is not independent of the gas compositions being compared and that the length of the evaporation area has a significant influence. Since the infinitely small evaporation from a small increment justifies the assumption of constant gas conditions in the direction of the gas flow, this inversion temperature can only be found by calculation. It is proposed to refer to this temperature as local inversion temperature. In contrast the gas conditions significantly change in the direction of the gas flow for finite evaporation surfaces as found in experimental investigations, so that only an apparent inversion temperature can be determined experimentally. This may explain the range of inversion temperatures reported. The calculation model is validated with experimental data from two independent previous experimental investigations. The calculation results are within ±20% of the experimental data. The calculations in this investigation do not involve any parameters fitted to the experimental data they are compared to, which clearly adds to the credibility of the model presented. Deviations are generally small and can be explained by the simplifying assumptions. It is shown that the apparent inversion temperature found for the constant rate of real drying processes may be significantly lower than the theoretically defined local value. The comparison of the evaporation rates on the basis of constant volumetric gas flows will lead to significantly higher values for the inversion temperature due to the difference in density of the gases.

Acknowledgements

This work was supported by a grant from the New Zealand Foundation for Research, Science and Technology under Contract WRO608 and by equipment and staff of the Wool Research Organisation of New Zealand. The author Jörg P Schwartze would like to thank A J McKinnon (WRONZ), H Höcker from the Deutsches Wollforschungsinstitut a.d. RWTH Aachen e.V. and R Keey from Canterbury University for their help and advice. Furthermore he would like to thank the German Academic Exchange Service DAAD for the financial support through a fellowship under the programme HSP III.

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